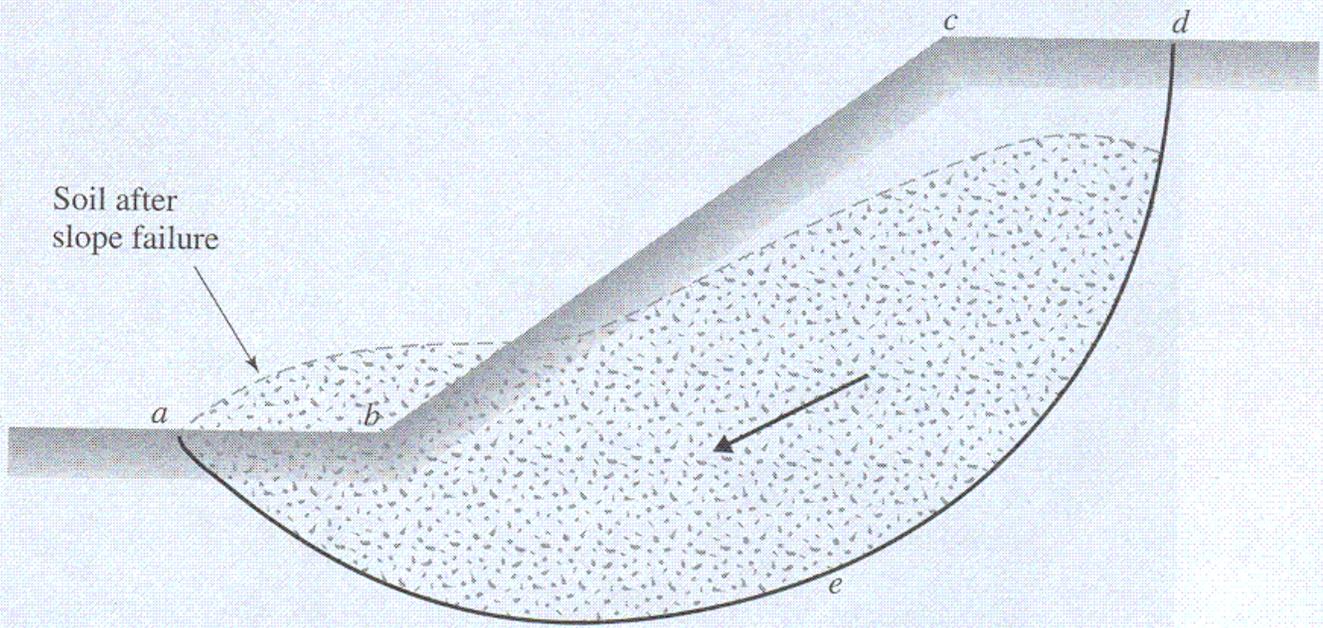


Advanced Soil Mechanics 61575

First Semester 2006-2007

Slope Stability

- **An exposed ground surface that stands at an angle with the horizontal is called an unrestrained slope.**
- **The slope can be natural or man-made.**
- **Gravity will tend to move the soil downward the slope as shown.**
- **If the component of gravity is large enough, slope failure can occur, that is, the soil mass in zone abcdea can slide downward as shown.**
- **The driving force overcomes the resistance from the shear strength of the soil along the rupture surface.**



(a)



(b)



(c)

- Civil engineers often are expected to make calculations to check the safety of natural slopes, slopes of excavations, and compacted embankments.
- This check involves determining the shear stress developed along the most likely rupture surface and comparing it with the shear strength of the soil.
- The above process is called slope stability analysis.
- The most likely rupture surface is the critical surface that has the minimum factor of safety.

- **The stability analysis of a slope is difficult to perform.**

Depends on:

- **Soil stratification**
- **In-place shear strength parameters**
- **Seepage through the slope**
- **Choice of a potential slip surface**

Factor of Safety

- The task of the engineer is to determine the factor of safety when analyzing slope stability.

Factor of Safety

The task of the engineer charged with analyzing slope stability is to determine the factor of safety. Generally, the factor of safety is defined as

$$F_s = \frac{\tau_f}{\tau_d} \quad (14.1)$$

where F_s = factor of safety with respect to strength

τ_f = average shear strength of the soil

τ_d = average shear stress developed along the potential failure surface

The shear strength of a soil consists of two components, cohesion and friction, and may be written as

$$\tau_f = c' + \sigma' \tan \phi' \quad (14.2)$$

where c' = cohesion

ϕ' = angle of friction

σ' = normal stress on the potential failure surface

In a similar manner, we can write

$$\tau_d = c'_d + \sigma' \tan \phi'_d \quad (14.3)$$

where c'_d and ϕ'_d are, respectively, the cohesion and the angle of friction that develop along the potential failure surface. Substituting Eqs. (14.2) and (14.3) into Eq. (14.1), we get

$$F_s = \frac{c' + \sigma' \tan \phi'}{c'_d + \sigma' \tan \phi'_d} \quad (14.4)$$

Now we can introduce some other aspects of the factor of safety — that is, the factor of safety with respect to cohesion, F'_c , and the factor of safety with respect to friction, F'_{ϕ} . They are defined as

$$F_{c'} = \frac{c'}{c'_d} \quad (14.5)$$

and

$$F_{\phi'} = \frac{\tan \phi'}{\tan \phi'_d} \quad (14.6)$$

When we compare Eqs. (14.4) through (14.6), we can see that when $F_{c'}$ becomes equal to $F_{\phi'}$, it gives the factor of safety with respect to strength. Or, if

$$\frac{c'}{c'_d} = \frac{\tan \phi'}{\tan \phi'_d}$$

then we can write

$$F_s = F_{c'} = F_{\phi'} \quad (14.7)$$

When F_s is equal to 1, the slope is in a state of impending failure. Generally, a value of 1.5 for the factor of safety with respect to strength is acceptable for the design of a stable slope.

Assuming that the pore water pressure is zero, we will evaluate the factor of safety against a possible slope failure along a plane AB located at a depth H below the ground surface. The slope failure can occur by the movement of soil above the plane AB from right to left.

Let us consider a slope element $abcd$ that has a unit length perpendicular to the plane of the section shown. The forces, F , that act on the faces ab and cd are equal and opposite and may be ignored. The weight of the soil element is

$$W = (\text{Volume of soil element}) \times (\text{Unit weight of soil}) = \gamma LH \quad (14.8)$$

The weight W can be resolved into two components:

1. Force perpendicular to the plane $AB = N_a = W \cos \beta = \gamma LH \cos \beta$.
2. Force parallel to the plane $AB = T_a = W \sin \beta = \gamma LH \sin \beta$. Note that this is the force that tends to cause the slip along the plane.

Thus, the effective normal stress and the shear stress at the base of the slope element can be given, respectively, as

$$\sigma' = \frac{N_a}{\text{Area of base}} = \frac{\gamma LH \cos \beta}{\left(\frac{L}{\cos \beta}\right)} = \gamma H \cos^2 \beta \quad (14.9)$$

and

$$\tau = \frac{T_a}{\text{Area of base}} = \frac{\gamma LH \sin \beta}{\left(\frac{L}{\cos \beta}\right)} = \gamma H \cos \beta \sin \beta \quad (14.10)$$

The reaction to the weight W is an equal and opposite force R . The normal and tangential components of R with respect to the plane AB are

$$N_r = R \cos \beta = W \cos \beta \quad (14.11)$$

and

$$T_r = R \sin \beta = W \sin \beta \quad (14.12)$$

For equilibrium, the resistive shear stress that develops at the base of the element is equal to $(T_r)/(\text{Area of base}) = \gamma H \sin \beta \cos \beta$. The resistive shear stress may also be written in the same form as Eq. (14.3):

$$\tau_d = c'_d + \sigma' \tan \phi'_d$$

The value of the normal stress is given by Eq. (14.9). Substitution of Eq. (14.9) into Eq. (14.3) yields

$$\tau_d = c'_d + \gamma H \cos^2 \beta \tan \phi'_d \quad (14.13)$$

Thus,

$$\gamma H \sin \beta \cos \beta = c'_d + \gamma H \cos^2 \beta \tan \phi'_d$$

or

$$\begin{aligned} \frac{c'_d}{\gamma H} &= \sin \beta \cos \beta - \cos^2 \beta \tan \phi'_d \\ &= \cos^2 \beta (\tan \beta - \tan \phi'_d) \end{aligned} \quad (14.14)$$

The factor of safety with respect to strength has been defined in Eq. (14.7), from which we get

$$\tan \phi'_d = \frac{\tan \phi'}{F_s} \quad \text{and} \quad c'_d = \frac{c'}{F_s}$$

Substituting the preceding relationships into Eq. (14.14), we obtain

$$F_s = \frac{c'}{\gamma H \cos^2 \beta \tan \beta} + \frac{\tan \phi'}{\tan \beta} \quad (14.15)$$

For granular soils, $c' = 0$, and the factor of safety, F_s , becomes equal to $(\tan \phi')/(\tan \beta)$. This indicates that in an infinite slope in sand, the value of F_s is independent of the height H and the slope is stable as long as $\beta < \phi'$.

If a soil possesses cohesion and friction, the depth of the plane along which critical equilibrium occurs may be determined by substituting $F_s = 1$ and $H = H_{cr}$ into Eq. (14.15). Thus,

$$H_{cr} = \frac{c'}{\gamma \cos^2 \beta (\tan \beta - \tan \phi')} \quad (14.16)$$

Stability with Seepage

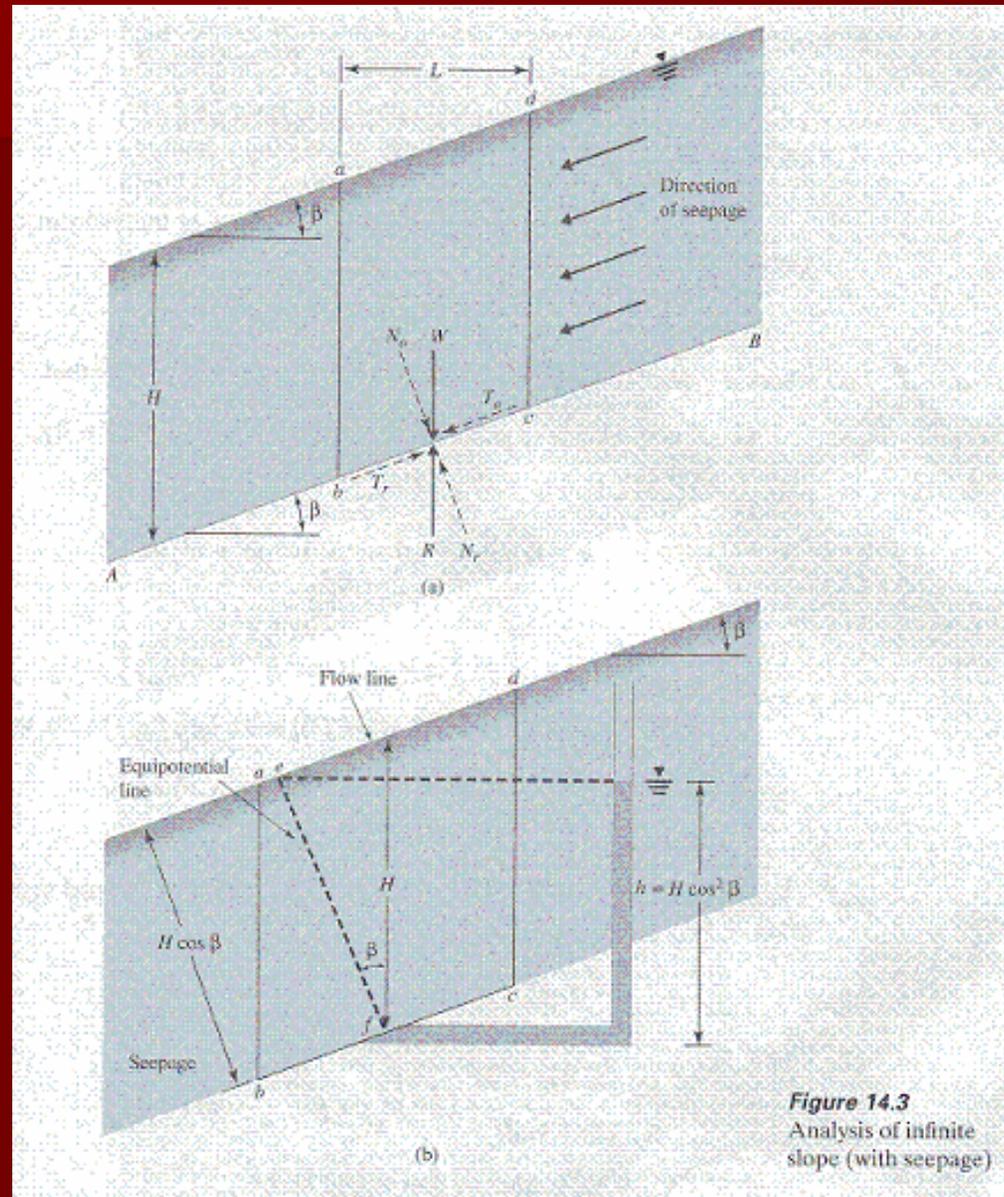


Figure 14.3
Analysis of infinite slope (with seepage)

Stability with Seepage Cont'd

$$F_s = \frac{c'}{\gamma_{\text{sat}} H \cos^2 \beta \tan \beta} + \frac{\gamma' \tan \phi'}{\gamma_{\text{sat}} \tan \beta}$$

(14.28)

Example 14.1

An infinite slope is shown in Figure 14.4. There is ground water seepage, and the ground water table coincides with the ground surface. Determine the factor of safety, F_s .

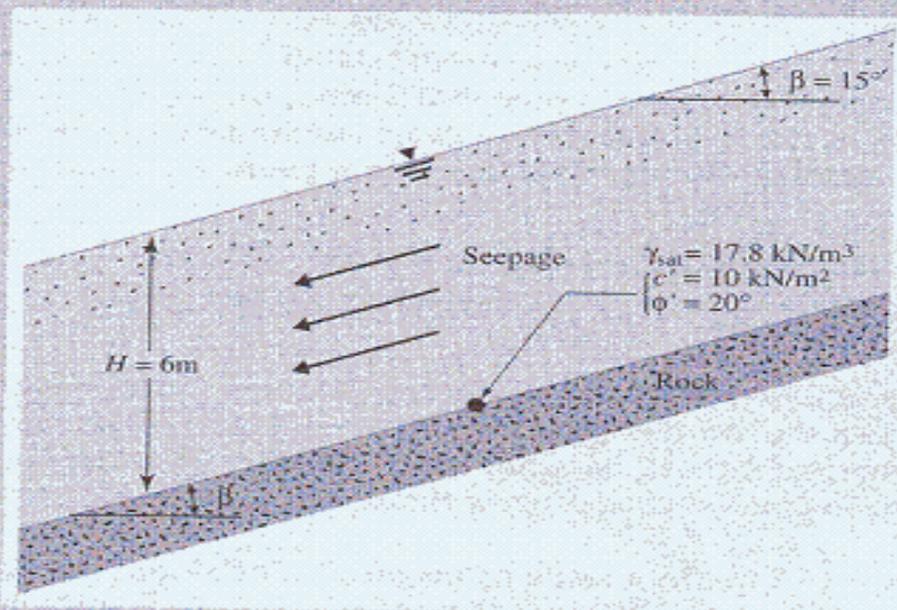


Figure 14.4

Solution

$$\gamma_{\text{sat}} = 17.8\text{ kN/m}^3 \quad \text{and} \quad \gamma_w = 9.81\text{ kN/m}^3$$

So,

$$\gamma' = \gamma_{\text{sat}} - \gamma_w = 17.8 - 9.81 = 7.99\text{ kN/m}^3$$

From Eq. (14.28),

$$\begin{aligned} F_s &= \frac{c'}{\gamma_{\text{sat}} H \cos^2 \beta \tan \beta} + \frac{\gamma' \tan \phi'}{\gamma_{\text{sat}} \tan \beta} \\ &= \frac{10}{(17.8)(6)(\cos 15)^\2 \tan 15} + \frac{7.99 \tan 20}{17.8 \tan 15} \\ &= 0.375 + 0.61 = \mathbf{0.985} \end{aligned}$$

The value of F_s would be less than 1; hence, the slope would be unstable. ■

14.3 Finite Slopes - General

When the value of H_{cr} approaches the height of the slope, the slope may generally be considered finite. For simplicity, when analyzing the stability of a finite slope in a homogeneous soil, we need to make an assumption about the general shape of the surface of potential failure. Although considerable evidence suggests that slope failures usually occur on curved failure surfaces, Culmann (1875) approximated the surface of potential failure as a plane. The factor of safety, F_s , calculated by using Culmann's approximation, gives fairly good results for near-vertical slopes only. After extensive investigation of slope failures in the 1920s, a Swedish geotechnical commission recommended that the actual surface of sliding may be approximated to be circularly cylindrical.

Since that time, most conventional stability analyses of slopes have been made by assuming that the curve of potential sliding is an arc of a circle. However, in many circumstances (for example, zoned dams and foundations on weak strata), stability analysis using plane failure of sliding is more appropriate and yields excellent results.

Analysis of Finite slopes with Plane Failure Surfaces (Culmann's Method)

Culmann's analysis is based on the assumption that the failure of a slope occurs along a plane when the average shearing stress tending to cause the slip is more than the shear strength of the soil. Also, the most critical plane is the one that has a minimum ratio of the average shearing stress that tends to cause failure to the shear strength of soil.

Figure 14.5 shows a slope of height H . The slope rises at an angle β with the horizontal. AC is a trial failure plane. If we consider a unit length perpendicular to the section of the slope, we find that the weight of the wedge ABC

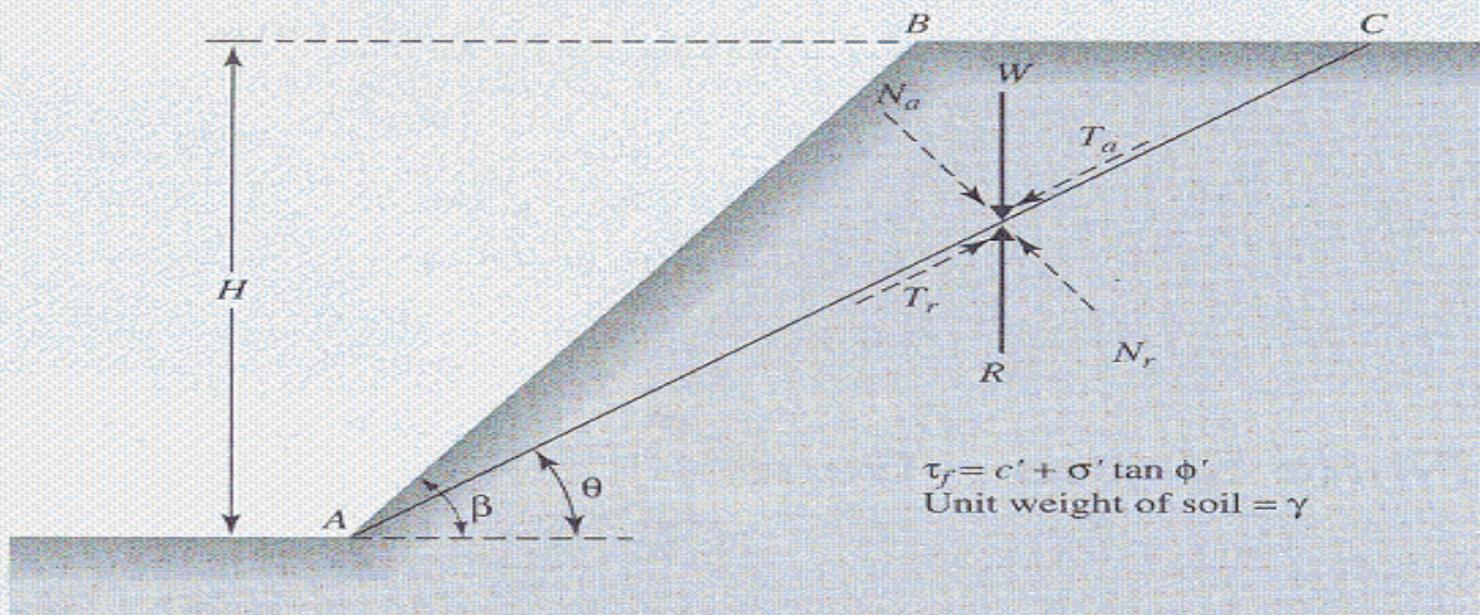


Figure 14.5 Finite slope analysis — Culmann's method

$$H = \frac{4c_d}{\gamma}$$

$$\frac{\sin \beta \cos \phi'_d}{1 - \cos(\beta - \phi'_d)}$$



$$c'_d = \frac{\gamma H}{4} \left[\frac{1 - \cos(\beta - \phi'_d)}{\sin \beta \cos \phi'_d} \right]$$

(14.40)

The preceding equation can also be written as

$$\frac{c'_d}{\gamma H} = m = \frac{1 - \cos(\beta - \phi'_d)}{4 \sin \beta \cos \phi'_d}$$

(14.41)

where m = stability number.

The maximum height of the slope for which critical equilibrium occurs can be obtained by substituting $c'_d = c'$ and $\phi'_d = \phi'$ into Eq. (14.40). Thus,

$$H_{cr} = \frac{4c'}{\gamma} \left[\frac{\sin \beta \cos \phi'}{1 - \cos(\beta - \phi')} \right]$$

(14.42)

Example 14.2

A cut is to be made in a soil that has $\gamma = 16 \text{ kN/m}^3$, $c' = 28 \text{ kN/m}^2$, and $\phi' = 20^\circ$. The side of the cut slope will make an angle of 45° with the horizontal. What should be the depth of the cut slope that will have a factor of safety, F_s , of 3.5?

Solution

We are given that $\phi' = 20^\circ$ and $c' = 28 \text{ kN/m}^2$. If $F_s = 3.5$, then, from Eq. (14.7), $F_{c'}$ and $F_{\phi'}$ should both be equal to 3.5. From Eq. (14.5),

$$F_{c'} = \frac{c'}{c'_d}$$

or

$$c'_d = \frac{c'}{F_{c'}} = \frac{c'}{F_s} = \frac{28}{3.5} = 8 \text{ kN/m}^2$$

Similarly, from Eq. (14.6),

$$F_{\phi'} = \frac{\tan \phi'}{\tan \phi'_d}$$

$$\tan \phi'_d = \frac{\tan \phi'}{F_{\phi'}} = \frac{\tan \phi'}{F_s} = \frac{\tan 20}{3.5}$$

OR

$$\phi'_d = \tan^{-1} \left[\frac{\tan 20}{3.5} \right] = 5.9^\circ$$

Substituting the preceding values of c'_d and ϕ'_d into Eq. (14.40) gives

$$\begin{aligned} H &= \frac{4c'_d}{\gamma} \left[\frac{\sin \beta \cos \phi'_d}{1 - \cos(\beta - \phi'_d)} \right] = \frac{4 \times 8}{16} \left[\frac{\sin 45 \cos 5.9}{1 - \cos(45 - 5.9)} \right] \\ &= 6.28 \text{ m} \end{aligned}$$

Analysis of Finite Slopes with Circular Failure Surface – General

Modes of Failure

In general, finite slope failure occurs in one of the following modes (Figure 14.6):

1. When the failure occurs in such a way that the surface of sliding intersects the slope at or above its toe, it is called a *slope failure* (Figure 14.6a). The failure circle is referred to as a *toe circle* if it passes through the toe of the slope and as a *slope circle* if it passes above the toe of the slope. Under certain circumstances, a *shallow slope failure* can occur, as shown in Figure 14.6b.
2. When the failure occurs in such a way that the surface of sliding passes at some distance below the toe of the slope, it is called a *base failure* (Figure 14.6c). The failure circle in the case of base failure is called a *midpoint circle*.

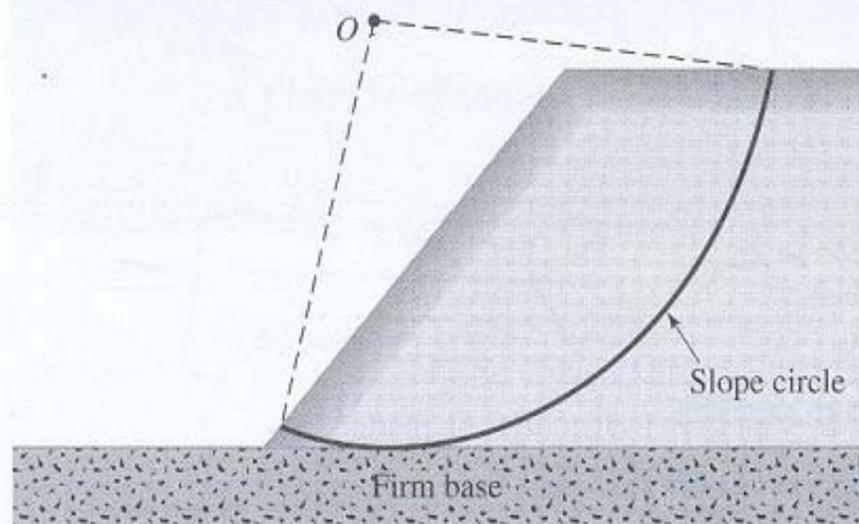
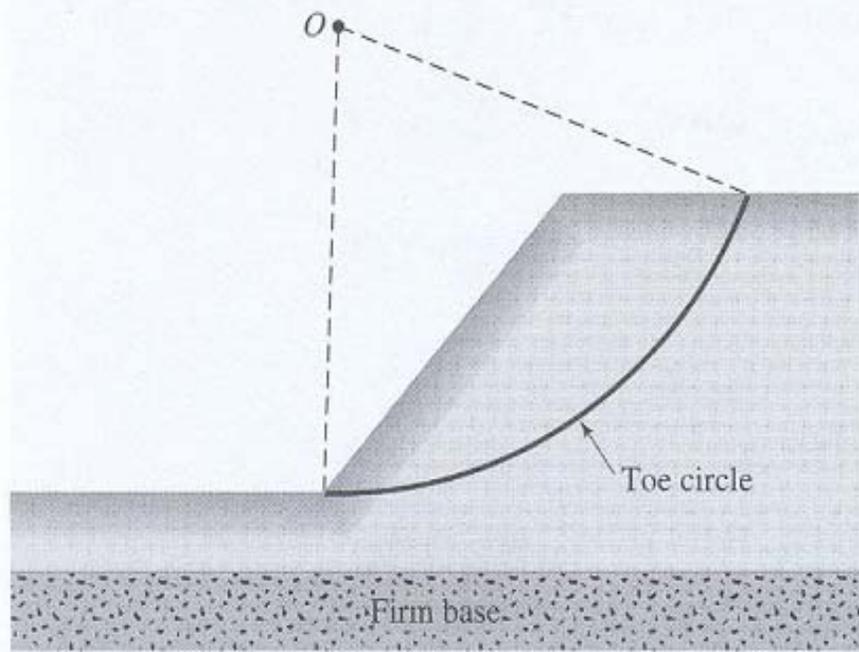
Types of Stability Analysis Procedures

Various procedures of stability analysis may, in general, be divided into two major classes:

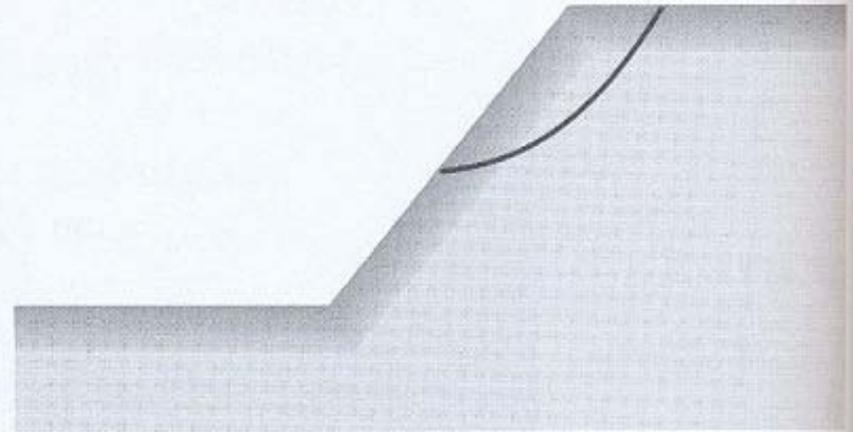
1. *Mass procedure*: In this case, the mass of the soil above the surface of sliding is taken as a unit. This procedure is useful when the soil that forms the slope is assumed to be homogeneous, although this is not the case in most natural slopes.
2. *Method of slices*: In this procedure, the soil above the surface of sliding is divided into a number of vertical parallel slices. The stability of each slice is calculated separately. This is a versatile technique in which the nonhomogeneity

of the soils and pore water pressure can be taken into consideration. It also accounts for the variation of the normal stress along the potential failure surface.

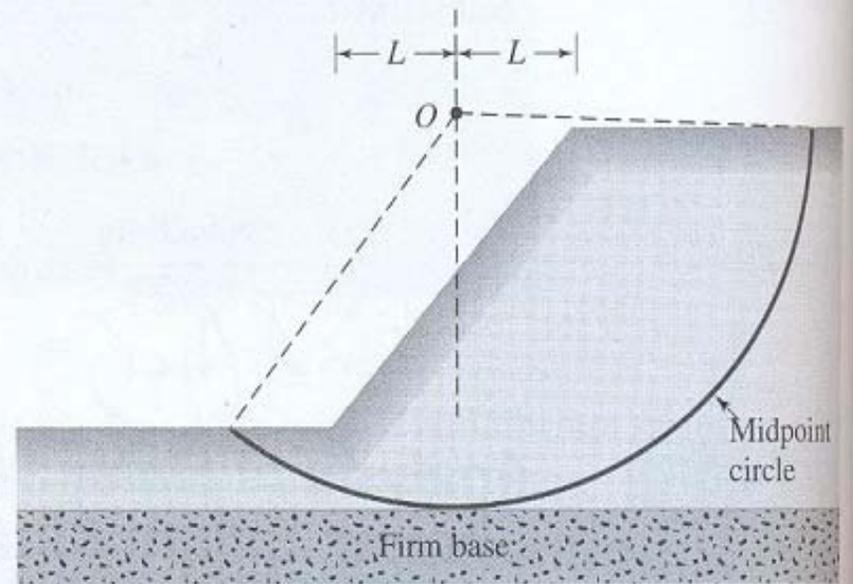
The fundamentals of the analysis of slope stability by mass procedure and method of slices are given in the following sections.



(a)



(b)



(c)

Figure 14.6 Modes of failure of finite slope: (a) slope failure; (b) shallow slope failure; (c) base failure

Mass Procedure – Slopes in Homogeneous Clay Soil with $\phi = 0$

Figure 14.7 shows a slope in a homogeneous soil. The undrained shear strength of the soil is assumed to be constant with depth and may be given by $\tau_f = c_u$. To perform the stability analysis, we choose a trial potential curve of sliding, AED , which is an arc of a circle that has a radius r . The center of the circle is located at O . Considering a unit length perpendicular to the section of the slope, we can give the weight of the soil above the curve AED as $W = W_1 + W_2$, where

$$W_1 = (\text{Area of } FCDEF)(\gamma)$$

and

$$W_2 = (\text{Area of } ABFEA)(\gamma)$$

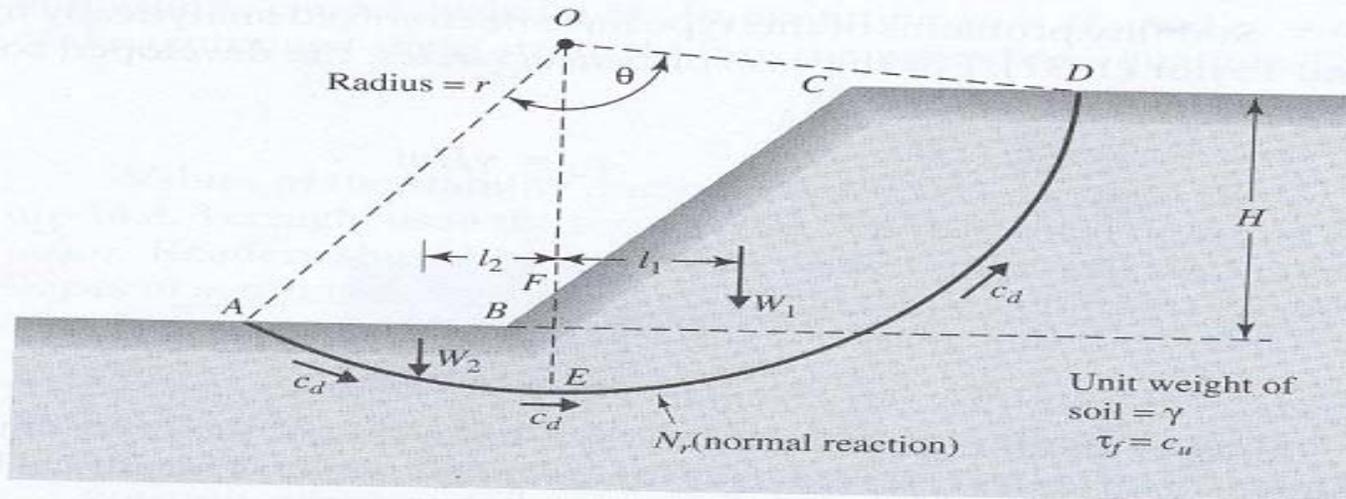


Figure 14.7 Stability analysis of slope in homogeneous saturated clay soil ($\phi = 0$)

Failure of the slope may occur by sliding of the soil mass. The moment of the driving force about O to cause slope instability is

$$M_d = W_1 l_1 - W_2 l_2 \quad (14.43)$$

where l_1 and l_2 are the moment arms.

The resistance to sliding is derived from the cohesion that acts along the potential surface of sliding. If c_d is the cohesion that needs to be developed, the moment of the resisting forces about O is

$$M_R = c_d (\widehat{AED})(1)(r) = c_d r^2 \theta \quad (14.44)$$

For equilibrium, $M_R = M_d$; thus,

$$c_d r^2 \theta = W_1 l_1 - W_2 l_2$$

or

$$c_d = \frac{W_1 l_1 - W_2 l_2}{r^2 \theta} \quad (14.45)$$

The factor of safety against sliding may now be found:

$$F_s = \frac{\tau_f}{c_d} = \frac{c_u}{c_d} \quad (14.46)$$

Note that the potential curve of sliding, AED , was chosen arbitrarily. The critical surface is that for which the ratio of c_u to c_d is a minimum. In other words, c_d is maximum. To find the critical surface for sliding, one must make a number of trials for different trial circles. The minimum value of the factor of safety thus obtained is the factor of safety against sliding for the slope, and the corresponding circle is the critical circle.

Stability problems of this type have been solved analytically by Fellenius (1927) and Taylor (1937). For the case of *critical circles*, the developed cohesion can be expressed by the relationship

$$c_d = \gamma H m$$

or

$$\frac{c_d}{\gamma H} = m \quad (14.47)$$

Note that the term m on the right-hand side of the preceding equation is non-dimensional and is referred to as the *stability number*. The critical height (i.e., $F_s = 1$)

of the slope can be evaluated by substituting $H = H_{cr}$ and $c_d = c_u$ (full mobilization of the undrained shear strength) into the preceding equation. Thus,

$$H_{cr} = \frac{c_u}{\gamma m} \quad (14.48)$$

Values of the stability number, m , for various slope angles, β , are given in Figure 14.8. Terzaghi used the term $\gamma H/c_d$, the reciprocal of m and called it the stability factor. Readers should be careful in using Figure 14.8 and note that it is valid for slopes of saturated clay and is applicable to only undrained conditions ($\phi = 0$).

In reference to Figure 14.8, the following must be pointed out:

- ✓ 1. For a slope angle β greater than 53° , the critical circle is always a toe circle.
- ✓ 2. For $\beta < 53^\circ$, the critical circle may be a toe, slope, or midpoint circle, depending on the location of the firm base under the slope. This is called the depth function, which is defined as

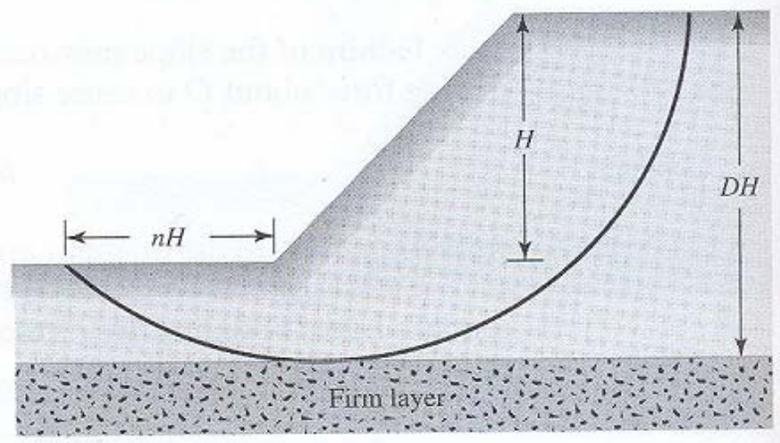
$$D = \frac{\text{Vertical distance from top of slope to firm base}}{\text{Height of slope}} \quad (14.49)$$

- ✓ 3. When the critical circle is a midpoint circle (i.e., the failure surface is tangent to the firm base), its position can be determined with the aid of Figure 14.9.
- ✓ 4. The maximum possible value of the stability number for failure as a midpoint circle is 0.181.

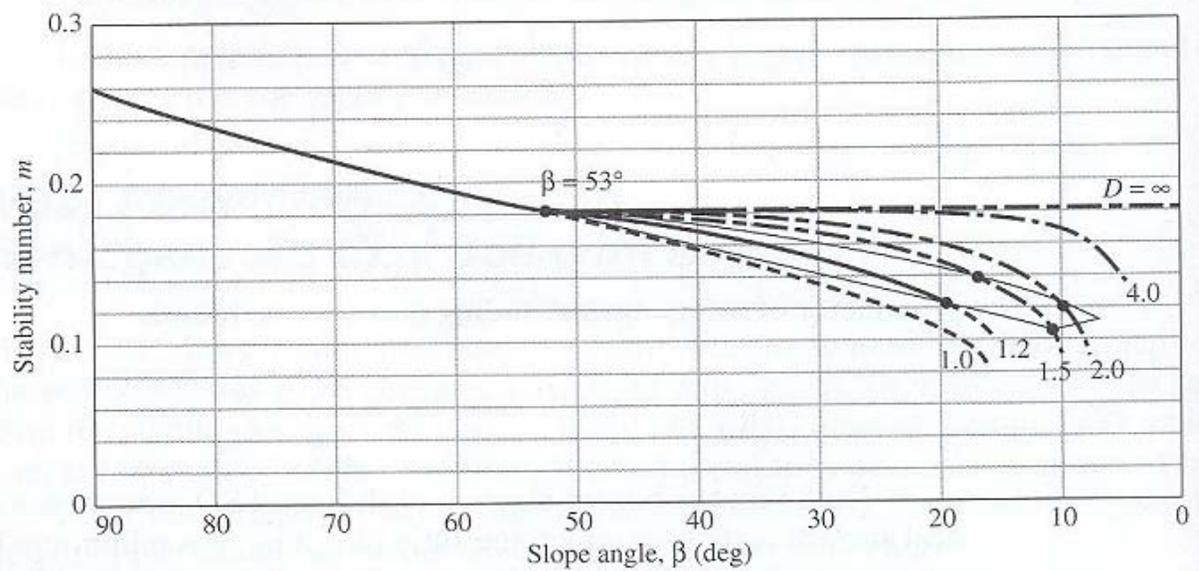
For $\beta > 53^\circ$:
 All circles are toe circles.

For $\beta < 53^\circ$:

Toe circle ————
 Midpoint circle - - - -
 Slope circle - - - -



(a)



(b)

Figure 14.8 (a) Definition of parameters for midpoint circle type of failure; (b) plot of stability number against slope angle (redrawn from Terzaghi and Peck, 1967)

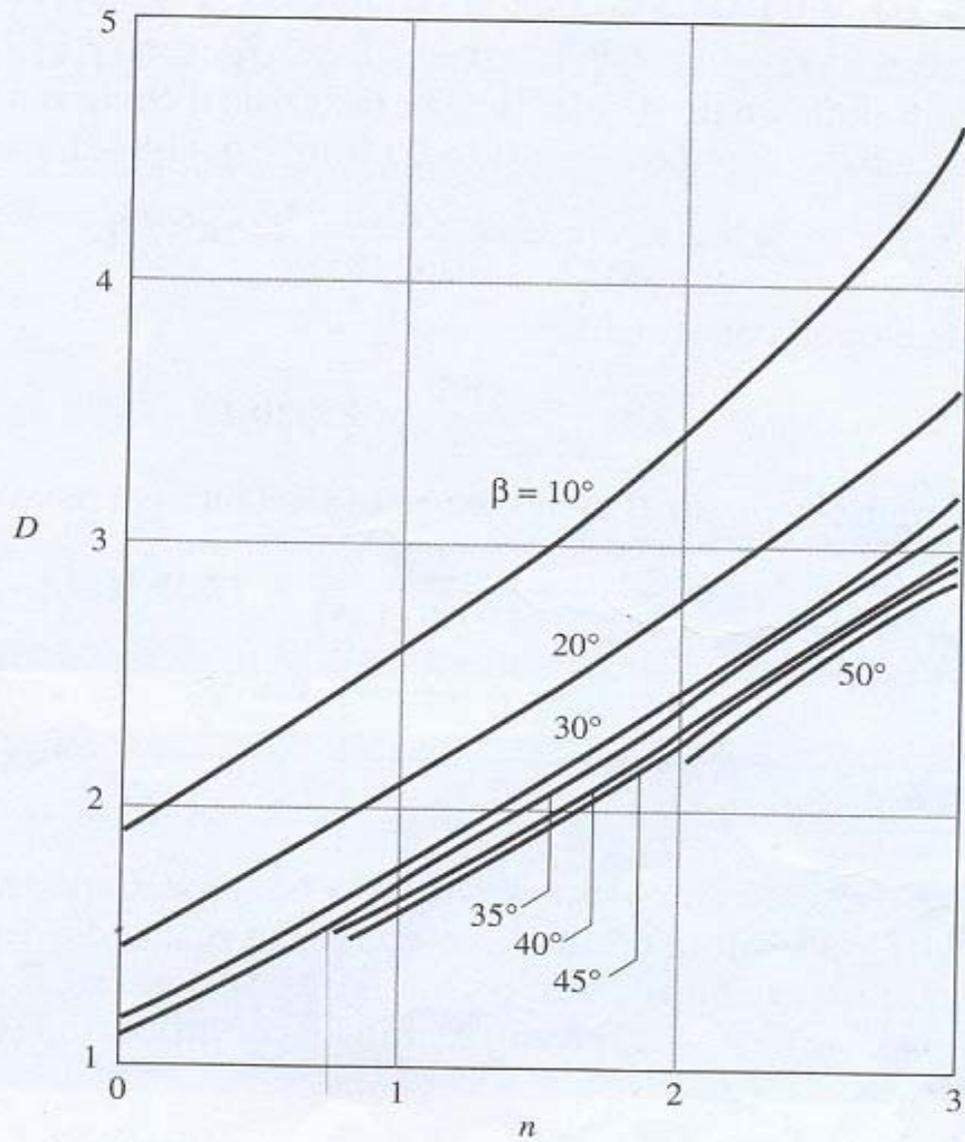


Figure 14.9
Location of midpoint circle

Example 14.3

A cut slope in saturated clay (Figure 14.10) makes an angle of 56° with the horizontal.

- Determine the maximum depth up to which the cut could be made. Assume that the critical surface for sliding is circularly cylindrical. What will be the nature of the critical circle (i.e., toe, slope, or midpoint)?
- How deep should the cut be made if a factor of safety of 2 against sliding is required?

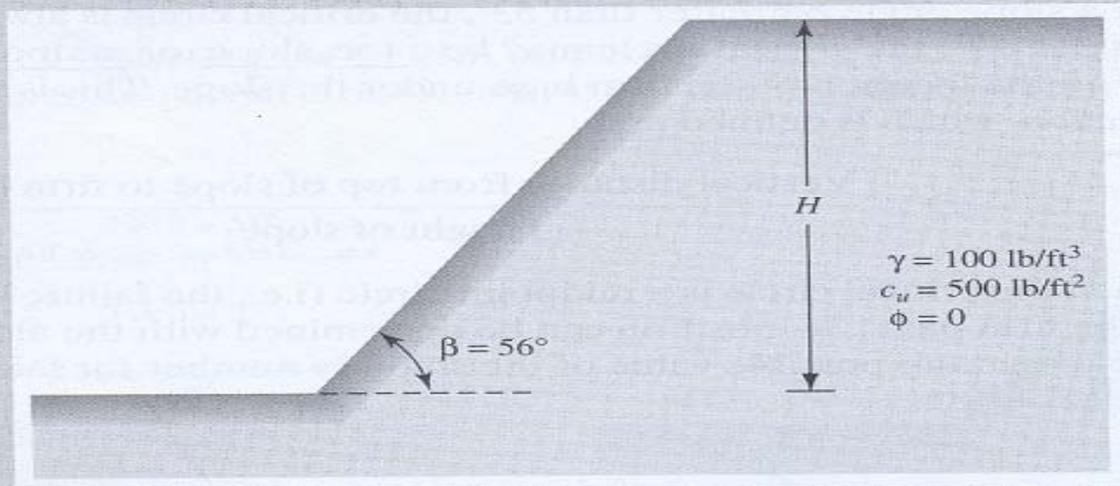


Figure 14.10

Solution

- Since the slope angle $\beta = 56^\circ > 53^\circ$, the critical circle is a **toe circle**. From Figure 14.8, for $\beta = 56^\circ$, $m = 0.185$. Using Eq. (14.48), we have

$$H_{cr} = \frac{c_u}{\gamma m} = \frac{500}{(110)(0.185)} = \mathbf{24.57 \text{ ft}}$$

- The developed cohesion is

$$c_d = \frac{c_u}{F_s} = \frac{500}{2} = 250 \text{ lb/ft}^2$$

From Figure 14.8, for $\beta = 56^\circ$, $m = 0.185$. Thus, we have

$$H = \frac{c_d}{\gamma m} = \frac{250}{(110)(0.185)} = \mathbf{12.29 \text{ ft}}$$

Example 14.4

A cut slope was excavated in a saturated clay. The slope made an angle of 40° with the horizontal. Slope failure occurred when the cut reached a depth of 6.1 m. Previous soil explorations showed that a rock layer was located at a depth of 9.15 m below the ground surface. Assume an undrained condition and $\gamma_{\text{sat}} = 17.29 \text{ kN/m}^3$.

- Determine the undrained cohesion of the clay (use Figure 14.8).
- What was the nature of the critical circle?
- With reference to the toe of the slope, at what distance did the surface of sliding intersect the bottom of the excavation?

Solution

- a. Referring to Figure 14.8, we find that

$$D = \frac{9.15}{6.1} = 1.5$$

$$\gamma_{\text{sat}} = 17.29 \text{ kN/m}^3$$

and

$$H_{cr} = \frac{c_u}{\gamma m}$$

- a. From Figure 14.8, for $\beta = 40^\circ$ and $D = 1.5$, $m = 0.175$, so

$$c_u = (H_{cr})(\gamma)(m) = (6.15)(17.29)(0.175) = \mathbf{18.6 \text{ kN/m}^2}$$

- b. **Midpoint circle**

- c. Again, from Figure 14.9, for $D = 1.5$, $\beta = 40^\circ$; $n = 0.9$, so distance = $(n)(H_{cr}) = (0.9)(6.1) = \mathbf{5.49 \text{ m}}$

Example 14.4

A cut slope was excavated in a saturated clay. The slope made an angle of 40° with the horizontal. Slope failure occurred when the cut reached a depth of 6.1 m. Previous soil explorations showed that a rock layer was located at a depth of 9.15 m below the ground surface. Assume an undrained condition and $\gamma_{\text{sat}} = 17.29 \text{ kN/m}^3$.

- Determine the undrained cohesion of the clay (use Figure 14.8).
- What was the nature of the critical circle?
- With reference to the toe of the slope, at what distance did the surface of sliding intersect the bottom of the excavation?

Solution

- Referring to Figure 14.8, we find that

$$D = \frac{9.15}{6.1} = 1.5$$

$$\gamma_{\text{sat}} = 17.29 \text{ kN/m}^3$$

and

$$H_{cr} = \frac{c_u}{\gamma m}$$

- From Figure 14.8, for $\beta = 40^\circ$ and $D = 1.5$, $m = 0.175$, so

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b. Midpoint circle

- Again, from Figure 14.9, for $D = 1.5$, $\beta = 40^\circ$; $n = 0.9$, so distance = $(n)(H_{cr}) = (0.9)(6.1) = \mathbf{5.49 \text{ m}}$

Mass Procedure—Slopes in Homogeneous c' - ϕ' Soil

A slope in a homogeneous soil is shown in Figure 14.14a. The shear strength of the soil is given by

$$\tau_f = c' + \sigma' \tan \phi'$$

The pore water pressure is assumed to be zero. \widehat{AC} is a trial circular arc that passes through the toe of the slope, and O is the center of the circle. Considering a unit length perpendicular to the section of the slope, we find

$$\text{Weight of soil wedge } ABC = W = (\text{Area of } ABC)(\gamma)$$

For equilibrium, the following other forces are acting on the wedge:

1. C_d —resultant of the cohesive force that is equal to the cohesion per unit area developed times the length of the cord \widehat{AC} . The magnitude of C_d is given by the following (Figure 13.17b):

$$C_d = c'_d(\widehat{AC}) \quad (14.53)$$

C_d acts in a direction parallel to the cord \widehat{AC} (see Figure 14.14b) and at a distance a from the center of the circle O such that

$$C_d(a) = c'_d(\widehat{AC})r$$

or

$$a = \frac{c'_d(\widehat{AC})r}{C_d} = \frac{\widehat{AC}}{AC}r \quad (14.54)$$

2. F —the resultant of the normal and frictional forces along the surface of sliding. For equilibrium, the line of action of F will pass through the point of intersection of the line of action of W and C_d .

Determination of the magnitude of c'_d described previously is based on a trial surface of sliding. Several trials must be made to obtain the most critical sliding surface, along which the developed cohesion is a maximum. Thus, we can express the maximum cohesion developed along the critical surface as

$$c'_d = \gamma H [f(\alpha, \beta, \theta, \phi')] \quad (14.55)$$

For critical equilibrium — that is, $F_c = F_{\phi'} = F_s = 1$ — we can substitute $H = H_{cr}$ and $c'_d = c'$ into Eq. (14.55) and write

$$c' = \gamma H_{cr} [f(\alpha, \beta, \theta, \phi')]$$

or

$$\frac{c'}{\gamma H_{cr}} = f(\alpha, \beta, \theta, \phi') = m \quad (14.56)$$

where m = stability number. The values of m for various values of ϕ' and β are given in Figure 14.15.

Calculations have shown that for $\phi > \sim 3^\circ$, the critical circles are all *toe circles*. Using Taylor's method of slope stability, Singh (1970) provided graphs of equal factors of safety, F_s , for various slopes. These graphs are given in Figure 14.16. In these charts, the pore water pressure is assumed to be zero.

The technique used to develop Figure 14.16 is illustrated in Example 14.7.

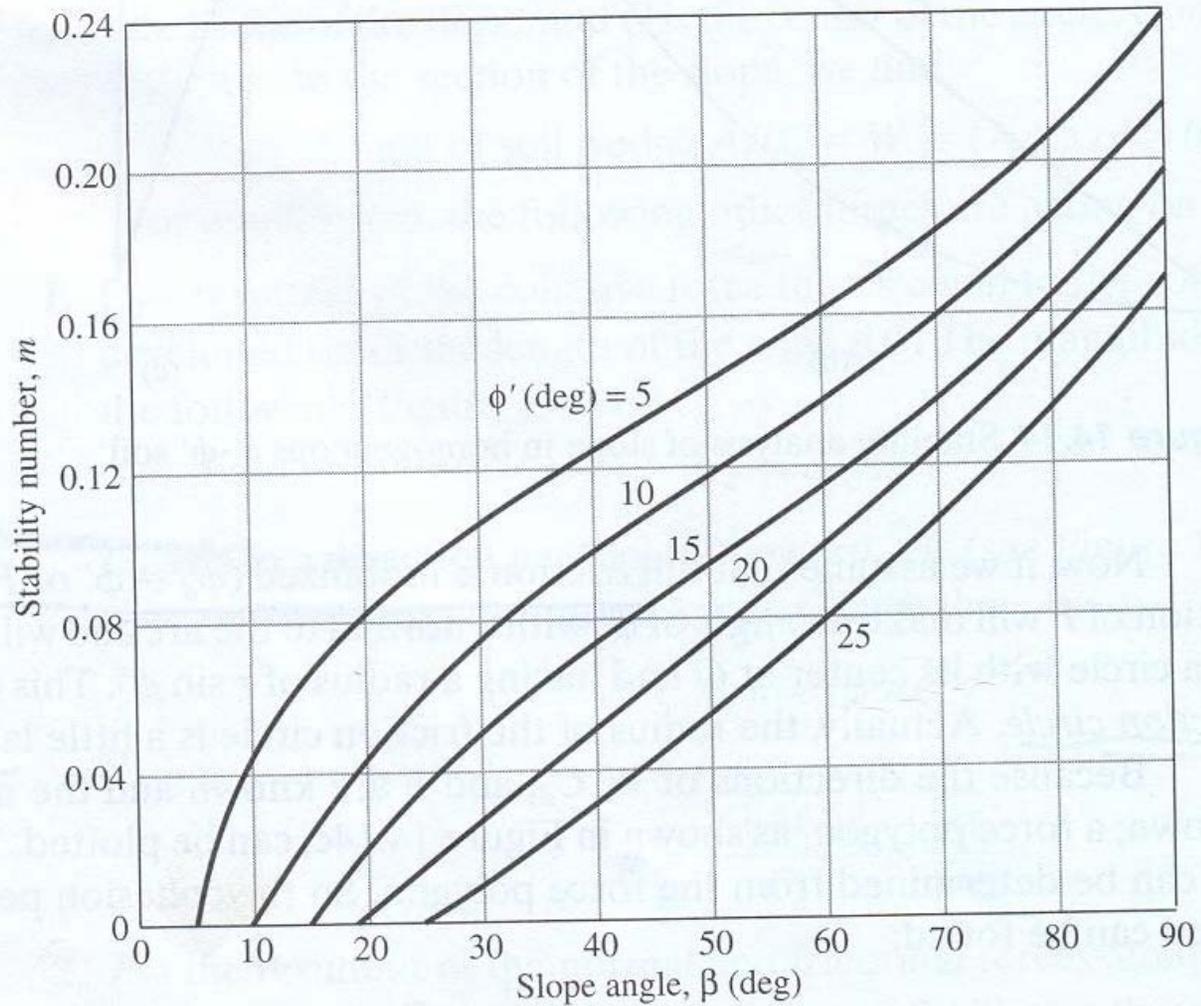


Figure 14.15 Plot of stability number with slope angle (after Taylor, 1937)

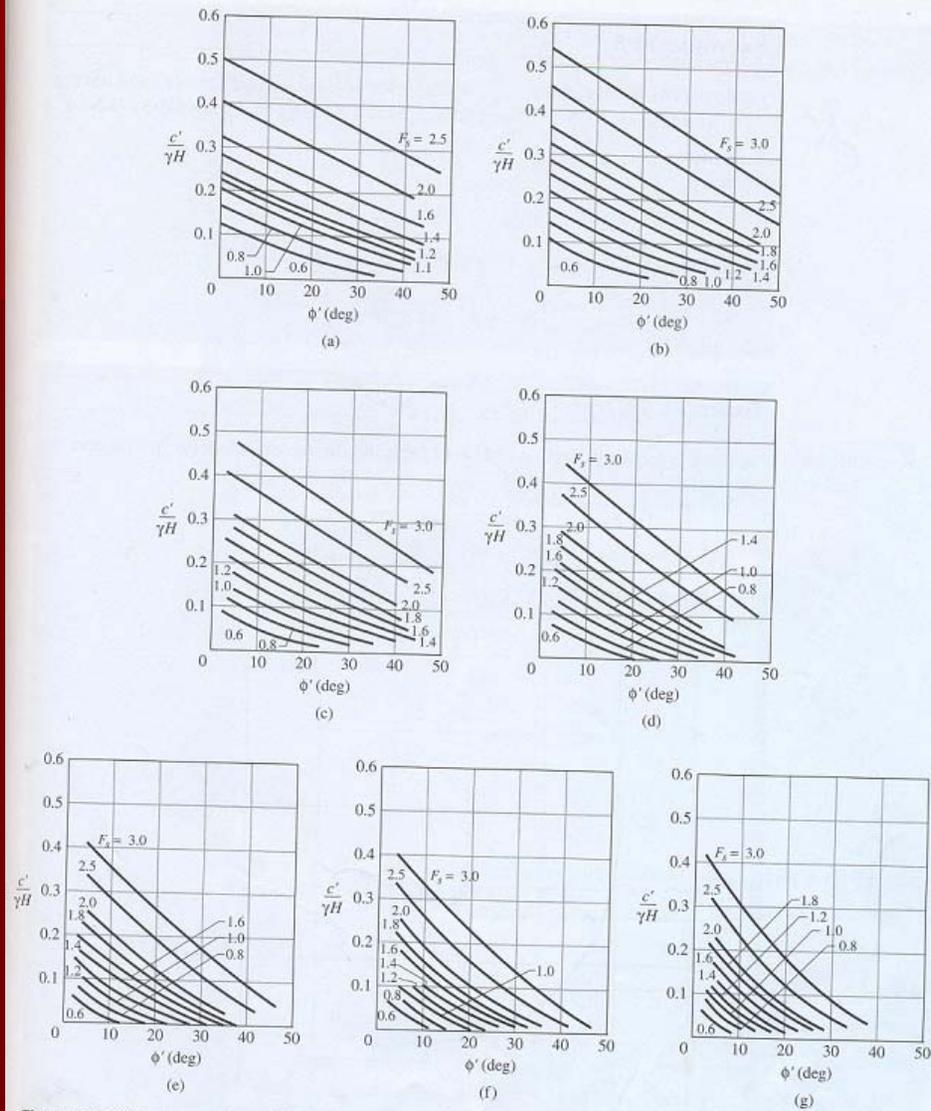


Figure 14.16 Contours of equal factors of safety: (a) slope - 1 vertical to 0.5 horizontal; (b) slope - 1 vertical to 0.75 horizontal; (c) slope - 1 vertical to 1 horizontal; (d) slope - 1 vertical to 1.5 horizontal; (e) slope - 1 vertical to 2 horizontal; (f) slope - 1 vertical to 2.5 horizontal; (g) slope - 1 vertical to 3 horizontal (after Singh, 1970)

Example 14.6

Find the critical height of a slope with $\beta = 45^\circ$ to be constructed with a soil having $\phi' = 20^\circ$ and $c' = 15 \text{ kN/m}^2$. The unit weight of the compacted soil will be 17 kN/m^3 .

Solution

We have

$$m = \frac{c'}{\gamma H_{cr}}$$

From Figure 14.15, for $\beta = 45^\circ$ and $\phi' = 20^\circ$, $m = 0.062$. So

$$H_{cr} = \frac{c'}{\gamma m} = \frac{15}{17 \times 0.062} = 14.2 \text{ m}$$

Example 14.7

A slope is shown in Figure 14.17a. Determine the factor of safety with respect to strength.

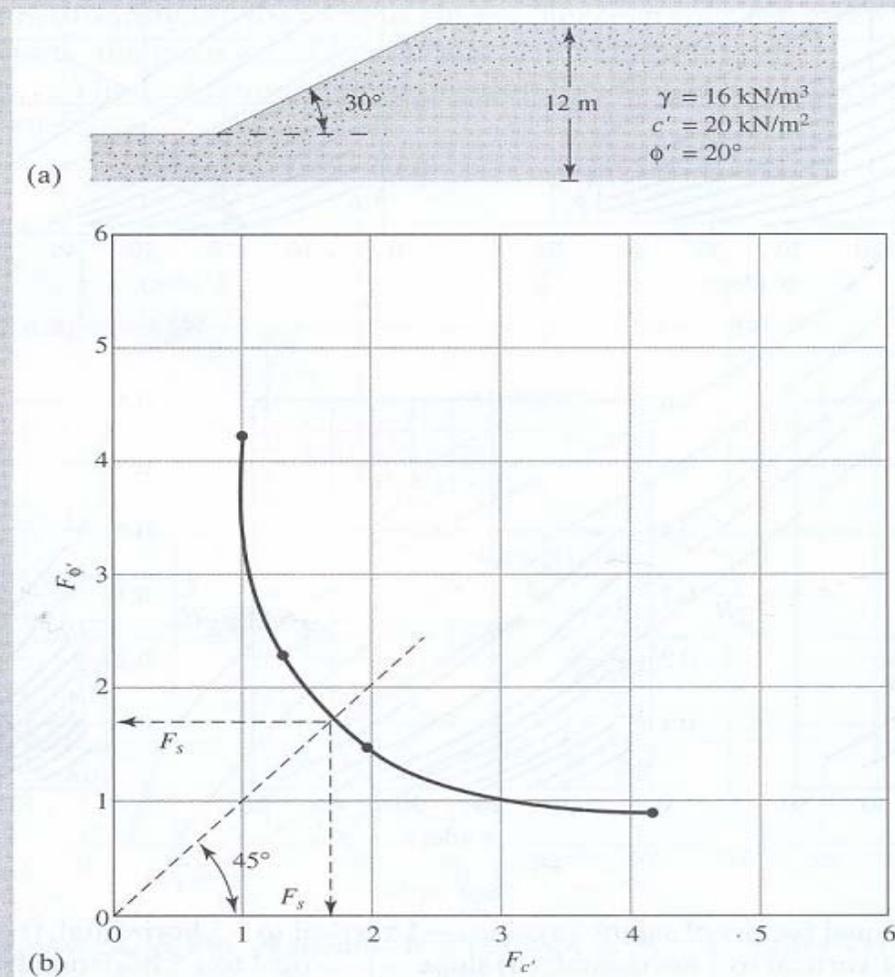


Figure 14.17

Solution

If we assume that full friction is mobilized, then, referring to Figure 14.15 (for $\beta = 30^\circ$ and $\phi'_d = \phi' = 20^\circ$), we obtain

$$m = 0.025 = \frac{c'_d}{\gamma H}$$

or

$$c'_d = (0.025)(16)(12) = 4.8 \text{ kN/m}^2$$

Thus,

$$F_{\phi'} = \frac{\tan \phi'}{\tan \phi'_d} = \frac{\tan 20}{\tan 20} = 1$$

and

$$F_c = \frac{c'}{c'_d} = \frac{20}{4.8} = 4.17$$

Since $F_c \neq F_{\phi'}$, this is not the factor of safety with respect to strength.

Now we can make another trial. Let the developed angle of friction, ϕ'_d , be equal to 15° . For $\beta = 30^\circ$ and the friction angle equal to 15° ,

$$m = 0.046 = \frac{c'_d}{\gamma H} \quad (\text{Figure 14.15})$$

or

$$c'_d = 0.046 \times 16 \times 12 = 8.83 \text{ kN/m}^2$$

For this trial,

$$F_{\phi'} = \frac{\tan \phi'}{\tan \phi'_d} = \frac{\tan 20}{\tan 15} = 1.36$$

and

$$F_c = \frac{c'}{c'_d} = \frac{20}{8.83} = 2.26$$

Similar calculations of $F_{\phi'}$ and F_c for various assumed values of ϕ'_d can be made and appear in the following table

ϕ'_d	$\tan \phi'_d$	$F_{\phi'}$	m	c'_d (kN/m ²)	F_c
20	0.364	1	0.025	4.8	4.17
15	0.268	1.36	0.046	8.83	2.26
10	0.176	2.07	0.075	14.4	1.39
5	0.0875	4.16	0.11	21.12	0.95

The values of $F_{\phi'}$ have been plotted against their corresponding values of F_c in Figure 14.17b, from which we get

$$F_c = F_{\phi'} = F_s = 1.73$$

14.9 Ordinary Method of Slices

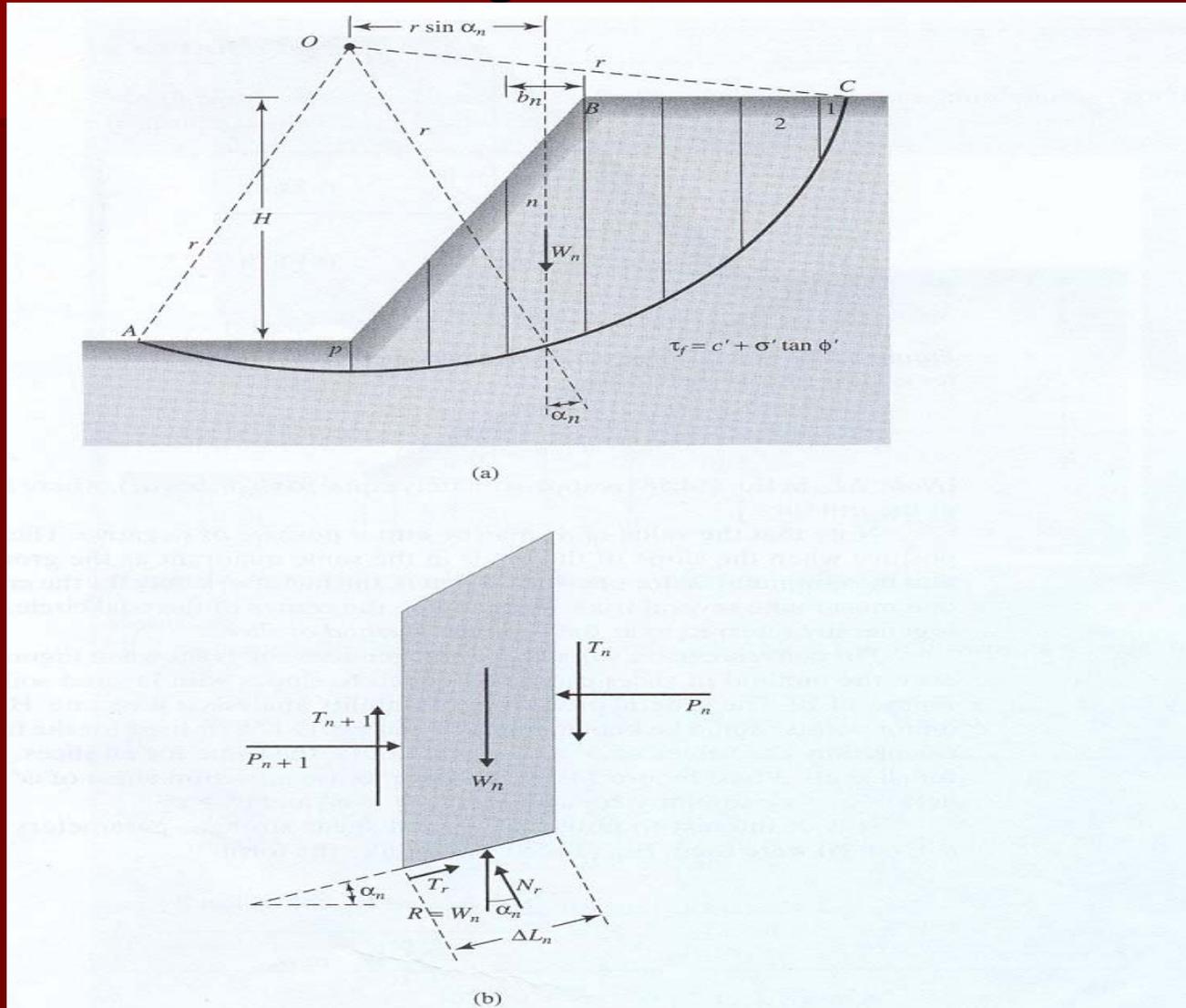


Figure 14.18 Stability analysis by ordinary method of slices: (a) trial failure surface; (b) forces acting on n th slice

$$F_s = \frac{\sum_{n=1}^{n=p} (c'\Delta L_n + W_n \cos \alpha_n \tan \phi')}{\sum_{n=1}^{n=p} W_n \sin \alpha_n} \quad (14.58)$$

Example 14.8

For the slope shown in Figure 14.20, find the factor of safety against sliding for the trial slip surface AC. Use the ordinary method of slices.

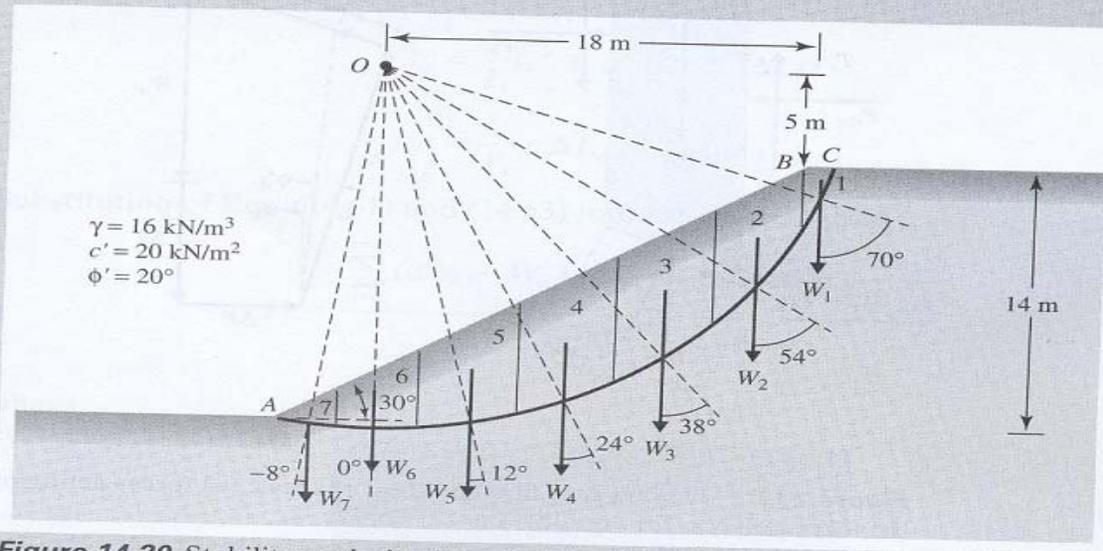


Figure 14.20 Stability analysis of a slope by ordinary method of slices

Solution

The sliding wedge is divided into seven slices. Now the following table can be prepared:

Slice no. (1)	W (kN/m) (2)	α_n (deg) (3)	$\sin \alpha_n$ (4)	$\cos \alpha_n$ (5)	ΔL_n (m) (6)	$W_n \sin \alpha_n$ (kN/m) (7)	$W_n \cos \alpha_n$ (kN/m) (8)
1	22.4	70	0.94	0.342	2.924	21.1	7.66
2	294.4	54	0.81	0.588	6.803	238.5	173.1
3	435.2	38	0.616	0.788	5.076	268.1	342.94
4	435.2	24	0.407	0.914	4.376	177.1	397.8
5	390.4	12	0.208	0.978	4.09	81.2	381.8
6	268.8	0	0	1	4	0	268.8
7	66.58	-8	-0.139	0.990	3.232	-9.25	65.9
					Σ Col. 6 =	Σ Col. 7 =	Σ Col. 8 =
					30.501 m	776.75 kN/m	1638 kN/m

$$F_s = \frac{(\Sigma \text{ Col. 6})(c') + (\Sigma \text{ Col. 8})\tan \phi'}{\Sigma \text{ Col. 7}}$$

$$= \frac{(30.501)(20) + (1638)(\tan 20)}{776.75} = 1.55$$

Bishop's Simplified Method of Slices

In 1955, Bishop proposed a more refined solution to the ordinary method of slices. In this method, the effect of forces on the sides of each slice are accounted for to some degree. We can study this method by referring to the slope analysis presented in Figure 14.18. The forces that act on the n th slice shown in Figure 14.18b have been redrawn in Figure 14.21a. Now, let $P_n - P_{n+1} = \Delta P$ and $T_n - T_{n+1} = \Delta T$. Also, we can write

$$T_r = N_r(\tan \phi'_d) + c'_d \Delta L_n = N_r \left(\frac{\tan \phi'}{F_s} \right) + \frac{c' \Delta L_n}{F_s} \quad (14.60)$$

Figure 14.21b shows the force polygon for equilibrium of the n th slice. Summing the forces in the vertical direction gives

$$W_n + \Delta T = N_r \cos \alpha_n + \left[\frac{N_r \tan \phi'}{F_s} + \frac{c' \Delta L_n}{F_s} \right] \sin \alpha_n$$

or

$$N_r = \frac{W_n + \Delta T - \frac{c' \Delta L_n}{F_s} \sin \alpha_n}{\cos \alpha_n + \frac{\tan \phi' \sin \alpha_n}{F_s}} \quad (14.61)$$

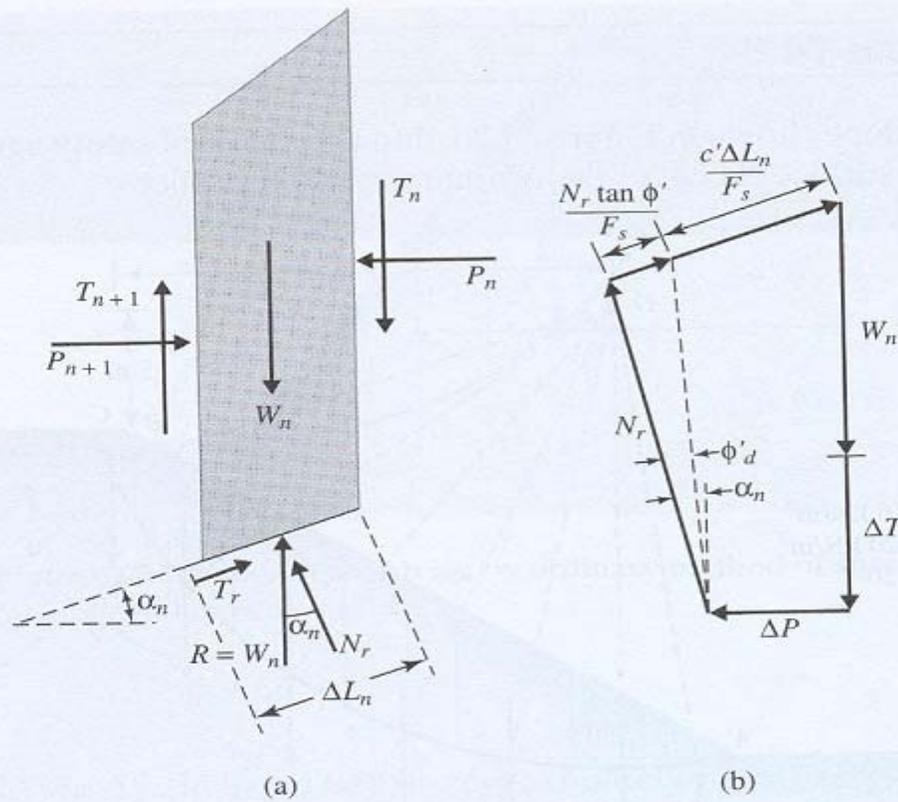


Figure 14.21 Bishop's simplified method of slices: (a) forces acting on the n th slice; (b) force polygon for equilibrium

For equilibrium of the wedge ABC (Figure 14.18a), taking the moment about O gives

$$\sum_{n=1}^{n=p} W_n r \sin \alpha_n = \sum_{n=1}^{n=p} T_r r \quad (14.62)$$

where

$$\begin{aligned} T_r &= \frac{1}{F_s} (c' + \sigma' \tan \phi') \Delta L_n \\ &= \frac{1}{F_s} (c' \Delta L_n + N_r \tan \phi') \end{aligned} \quad (14.63)$$

Substitution of Eqs. (14.61) and (14.63) into Eq. (14.62) gives

$$F_s = \frac{\sum_{n=1}^{n=p} (c' b_n + W_n \tan \phi' + \Delta T \tan \phi') \frac{1}{m_{\alpha(n)}}}{\sum_{n=1}^{n=p} W_n \sin \alpha_n} \quad (14.64)$$

where

$$m_{\alpha(n)} = \cos \alpha_n + \frac{\tan \phi' \sin \alpha_n}{F_s} \quad (14.65)$$

For simplicity, if we let $\Delta T = 0$, Eq. (14.64) becomes

$$F_s = \frac{\sum_{n=1}^{n=p} (c' b_n + W_n \tan \phi') \frac{1}{m_{\alpha(n)}}}{\sum_{n=1}^{n=p} W_n \sin \alpha_n} \quad (14.66)$$

Note that the term F_s is present on both sides of Eq. (14.66). Hence, we must adopt a trial-and-error procedure to find the value of F_s . As in the method of ordinary slices, a number of failure surfaces must be investigated so that we can find the critical surface that provides the minimum factor of safety.

Bishop's simplified method is probably the most widely used. When incorporated into computer programs, it yields satisfactory results in most cases. The ordinary method of slices is presented in this chapter as a learning tool only. It is rarely used now because it is too conservative.

